MATH 2050B Mathematical Analysis I
2023-24 Term 1
Problem Set 5
due on Oct 20, 2023 (Friday) at 11:59PM

Instructions: You are allowed to discuss with your classmates or seek help from the TAs but you are required to write/type up your own solutions. please do NOT come to campus to submit your completed assignments. Instead, you can either type up your assignment or scan a copy of your written assignment into ONE PDF file and submit through Gradescope on/before the due date. Please remember to write down your name and student ID. **No late homework will be accepted.** All the exercises below are taken from the textbook.

Required Readings: Chapter 3.2, 3.3

Optional Readings: none

Problems to hand in

Section 3.2: Exercise # 1(b)(c), 12, 18

Section 3.3: Exercise # 4, 6, 11

Suggested Exercises

Section 3.2: Exercise # 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 16, 17, 19, 23

Section 3.3: Exercise # 1, 2, 3, 5, 7, 8, 9, 10, 12

Challenging Exercises (optional)

1. Section 3.2: Exercise # 8, 15, 20, 21, 22
2. Let (x_n) be a sequence of real numbers. Define a new sequence (s_n) by
$$s_n := \frac{x_1 + x_2 + \cdots x_n}{n} \quad \text{for all } n \in \mathbb{N}.$$

(a) Show that $\lim(s_n) = x$ provided that $\lim(x_n) = x$.

(b) Find a divergent sequence (x_n) such that $\lim(s_n) = 0$.

3. Let $x_1 := \sqrt{2}$ and for $n \geq 2$,

$$x_n := \sqrt{2 + \sqrt{x_{n-1}}}.$$

Prove that (x_n) is convergent.

4. Let $a > \sqrt{2}$ be a fixed number. Define $x_1 := a$, and for $n \geq 2$,

$$x_n := \frac{2 + x_{n-1}}{1 + x_{n-1}}.$$

(a) Show that $x_1 > x_3 > x_5 > \cdots$.

(b) Show that $x_2 < x_4 < x_6 > \cdots$.

(c) Prove that $\lim(x_n) = \sqrt{2}$.